Few-Body Systems (EFB22) manuscript No.
(will be inserted by the editor)

Yubing Dong · Mauro Giannini · Elena Santopinto · Andrea Vassallo

Electromagnetic $N - \Delta(1232)$ transitions within the point-form of relativistic quantum mechanics

Received: date / Accepted: date

Abstract The electromagnetic $N-\Delta(1232)$ transition amplitudes are calculated using the point-form of relativistic quantum mechanics. The relativistic effects incorporated in the electromagnetic matrix elements give a good description of the transition amplitudes to the $\Delta(1232)$ resonance, reproducing well the Q^2 behaviour of the data, apart from the low Q^2 one.

Keywords Electromagnetic transition · $\Delta(1232)$ resonance · point-form of relativistic quantum mechanics · hyper-central potential model

1 Introduction

The study of nucleon electromagnetic form factors and the electromagnetic transitions of the nucleon resonances is always of great interest. It can give a detailed information on the internal structure of the nucleon and its excitations. There has been a large amount of model calculations in the past several decades, based on both the non-relativistic and relativistic frameworks. It is expected that more accurate data to a higher Q^2 region will come out with the 12 GeV upgraded JLab. facility. Therefore a more precise description of the transition amplitudes in this region is required.

In 1949, Dirac [1] first proposed three equivalent forms of the relativistic dynamics. They are the instant, light-front and point-forms. Here, we use the point-form, since all the components of the four-momentum P_μ ($\mu = 0, 1, 2, 3$) are associated with the interactions and other operators, like the angular momentum and Lorentz boost operators, are interaction free. Therefore, the advantage of the point-form of relativistic quantum mechanics is that all the Lorentz transformations remain purely kinematic and the theory is manifestly Lorentz covariant.

People are more familiar with the instant and light-front forms than the point-form, since the two were rather popular in the past decades and most of the calculations were based on the two frameworks. The point-form has been discussed by Keister and Polyzou [2] and recently has been carefully and systematically studied by Klink [3]. It has also been employed in the calculations of the nucleon form factors [4; 5; 6; 7; 8; 9], the resonance strong decays [10; 11], and the form factors of pion

This work is supported by National Sciences Foundations of China Nos. 10975146, 11035006 and 11261130, as well as supported, in part, by the DFG and the NSFC through funds provided to the Sino-Germen CRC 110 “Symmetries and the Emergence of Structure in QCD”.

Y. Dong
Institute of High Energy Physics, Beijing 100049, People’s Republic of China
Theoretical Physics Center for Science Facilities (TPCSF), CAS, Beijing 100049, People’s Republic of China
E-mail: dongyb@ihep.ac.cn

Mauro Giannini · Elena Santopinto · Andrea Vassallo
INFN, Sezione di Genova and universita di Genova, via Dodecaneso 33, I-16146, Genova, Italy

and deuteron [12; 13; 14; 15]. Those results show the importance of the relativistic description of the systems, particularly when the momentum transfer is large.

In this work, the point-form of relativistic quantum mechanics will be employed to calculate the electromagnetic transition amplitudes of the nucleon to $\Delta(1232)$. Here the wave functions of the nucleon and its resonances from the hyper-central potential model [16] are employed. It is expected that the relativistic description both for the wave functions and for the matrix elements could well reproduce the Q^2-dependence of the transition amplitudes. This work is organized as follows. In Sect. 2, the relativistic hyper-central potential model will be briefly discussed and the point-form of relativistic quantum mechanics is displayed and applied to the study of the electromagnetic $N - \Delta(1232)$ transitions. Numerical results and a short summary will be given in Sect 3.

2 Hyper-Central Potential Model and the Point-Form of Relativistic Quantum Mechanics

The hyper-central potential model was proposed a long time ago [16] and since then it has been used for the calculations of the baryon electromagnetic properties [17; 18; 19; 20; 21], in particular for the predictions of the transition form factors of the nucleon to its baryon resonances [22]. The model has also been extended to a relativistic version replacing the non-relativistic kinetic operator by a fully relativistic one [8; 9].

The mass operator in the relativistic hypercentral constituent quark model is given by [8; 9]

$$\hat{M} = \sum_{i=1}^{3} \sqrt{m^2 + \mathbf{k}_i{}^2} - \frac{\tau}{x} + \alpha x + M_{hyp}. \tag{1}$$

In our calculation, the center-of-mass frame is considered and thus $\sum_{i=1}^{3} \mathbf{k}_i = 0$. In Eq. (1), the hyper-radius $x = \sqrt{\boldsymbol{\rho}^{\,2} + \boldsymbol{\lambda}^{\,2}}$ with $\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2)$ and $\boldsymbol{\lambda} = \frac{1}{\sqrt{6}}(\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)$ being the internal Jacobi coordinates. M_{hyp} is the hyperfine interaction, which is spin-dependent. The spin-independent part of the interaction includes, at least in some sense, the three-body interactions. It is different from the other ordinary constituent quark models where only the two-body interactions are taken into account. On the other hand, it may be considered as the hypercentral approximation to the two-body potential. The relativistic mass operator can be diagonalized by means of a variational method and one has to work in the momentum space due to the relativistic kinetic energy operator.

In the point-form of relativistic quantum mechanics, in order to construct the interacting four-momentum operator, one usually uses the Bakamjian-Thomas method [23] by putting the interactions into the mass operator $\hat{\mathcal{M}}$. Thus, $\hat{\mathcal{M}}$ is divided into two parts. One is the interaction free mass operator $\hat{\mathcal{M}}_{fr}$ and another is the interacting mass operator $\hat{\mathcal{M}}_{int}$. The four-momentum P^μ is related to the mass operator by

$$P^\mu = \hat{\mathcal{M}} V^\mu_{fr}, \tag{2}$$

where the four-velocity operator V^μ_{fr} is interaction free. According to the commutation relations satisfied by the operators of the dynamical system and to the fact that P^μ is a Lorentz vector, one gets the relation of $[V^\mu_{fr}, \hat{\mathcal{M}}] = 0$ and $\hat{\mathcal{M}}$ is a Lorentz scalar. Therefore, the eigenstates of the four-momentum operator are the eigenstates of both the mass and the velocity operators. In the center-of-mass frame, we can obtain the wave functions of the three-quark system by solving a relativistic Schrödinger equation. The obtained wave functions are the eigenstates of the mass operator with interactions.

In the point-form of relativistic quantum mechanics, the Lorentz transformations remain purely kinematic, namely, they are interaction free. The so-called velocity state is usually introduced as follows [3],

$$\begin{aligned} \mid v; \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \mu_1, \mu_2, \mu_3 > &= U_{B(v)} \mid \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \mu_1, \mu_2, \mu_3 > \\ &= \Pi_{i=1}^{3} D^{1/2}_{\sigma_i \mu_i} \left[R_W(k_i, B(v)\right] \mid \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \sigma_1, \sigma_2, \sigma_3 > \end{aligned} \tag{3}$$

where k_i (with $i = 1, 2, 3$) are the quark momenta in the center-of-mass system, $B(v)$ is a Lorentz boost with four-velocity v, $p_i = B(v)k_i$, and $U_{B(v)}$ is a unitary representation of $B(v)$. $D^{1/2}(RW)$ is the spin-1/2 representation matrix of the Wigner rotation. It has been proved [3] that all the Wigner rotations

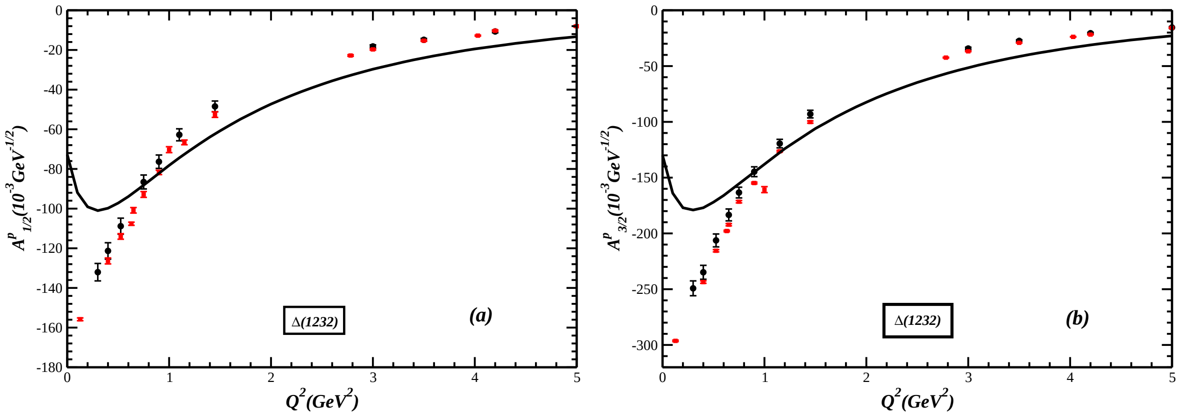

Fig. 1 The preliminarly estimated $N - \Delta(1232)$ transverse transition amplitudes (solid curves), (a) for $A_{1/2}$ and (b) for $A_{3/2}$. The date are from Refs. [24; 25].

of a canonical boost of a velocity state are the same, and thus the spins can be coupled together to the total spin of the state as in the non-relativistic framework as well as in the center-of-mass frame. This is the practical advantage of using the velocity state.

To calculate the photo- and electro-production amplitudes of the nucleon resonances, we simply employ the point-form spectator impulse approximation for the electromagnetic interaction. The current operator is assumed to be the single-particle one [4; 5; 6; 7],

$$< p_i', \lambda_i' \mid j^\mu \mid p_i, \lambda_i >= e_i \bar{u}(p_i', \lambda_i')\gamma^\mu u(p_i, \lambda_i), \tag{4}$$

where $u(p_i, \lambda_i)$ is the Dirac spinor with momentum p_i and spin λ_i for the i-th struck quark.

3 Numerical results and summary

In this work, we present the preliminary results for the electromagnetic transition amplitudes of the $N - \Delta(1232)$ based on the point-form of relativistic quantum mechanics. Here we employ the wave functions of the nucleon and the nucleon resonance $\Delta(1232)$ obtained from the relativistic hyper-central potential model. Figure 1 reports the obtained transverse transition amplitudes to the $\Delta(1232)$. In the figure the data, from Refs. [24; 25], are also shown for a comparison. We can see that our present framework can well reproduce the transverse transition amplitudes of the $\Delta(1232)$ resonance in the region of $Q^2 > \ 1\ GeV^2$. The present calculation with the point-form of relativistic quantum mechanics is an improvement with respect to the calculations with the non-relativistic hyper-central potential model. For the amplitudes in the small Q^2 region, it is expected that the quark-antiquark pair production mechanism plays a dominant role. In order to take into account this effect in a consistent way one has to work within the unquenched quark model [26; 27; 28].

To summarize, we have applied the wave functions of the nucleon and $\Delta(1232)$, obtained from the relativistic hyper-central potential model, for the calculation of the electromagnetic $N - \Delta(1232)$ transition amplitudes based on the point-form of relativistic quantum mechanics. Our numerical results show explicitly the advantage of the present fully relativistic description in the region of $Q^2 > 1GeV^2$. Other electromagnetic observables of the nucleon resonances, like the transition amplitudes to the low-lying nucleon resonances of $S_{11}(1535)$ and $D_{13}(1520)$, will be published elsewhere.

Acknowledgements One of Author (YBD) thanks the Department of Physics, Genova University, for the warm hospitality.

References

1. Dirac, P. A. M.: Forms of Relativistic Dynamics. Rev. Mod. Phys. **21**, 392-399 (1949)

2. Keister, B. D, Polyzou, W.N.: In: Negele, J. W, Vogt, E. W. (eds.) Advanced Nuclear Physics, vol. **21** pp.225-479. Plenum (1991)
3. Klink, W. H.: Point form relativistic quantum mechanics and electromagnetic form factors. Phys. Rev. C. **58** 3587-3604, (1998)
4. Wagenbrunn, R. F., Boffi, S., Klink, W., Plessas, W., Radici, R.: Covariant nucleon electromagnetic form-factors from the Goldstone boson exchange quark model. Phys. Lett. B. **511**, 33-39 (2001)
5. Glozman, L. Y., Radici, M., Wagenbrunn, R. F., Boffi, S., Klink, W., Plessas, W.: Covariant axial form-factor of the nucleon in a chiral constituent quark model. Phys. Lett. B. **516**, 183-190 (2001)
6. Boffi, S., Glozman, L. Y., Klink, W., Plessas, W., Radici, M., Wagenbrunn, R. F.: Covariant electroweak nucleon form-factors in a chiral constituent quark model. Eur. Phys. J. A. **14**, 17-21 (2002)
7. Melde, T., Berger, K., Canton, L., Plessas, W., Wagenbrum, R. F.: Electromagnetic nucleon form factors in instant and point form. Phys. Rev. D. **76**, 074020 (2007)
8. De Sanctis, M., Giannini, M. M., Santopinto, E., Vassallo, A.: Electromagnetic form factors and the hypercentral constituent quark model. Phys. Rev. C. **76**, 062201 (2007)
9. Santopinto, E., Vassallo, A., Giannini, M. M., De Sanctis, M.: High Q^2 behavior of the electromagnetic form factors in the relativistic hypercentral constituent quark model. Phys. Rev. C. **82** 065204 (2010)
10. Melde, T., Plessas, W., Wagenbrunn, R. F.: Covariant calculation of mesonic baryon decays. Phys. Rev. C. **72**, 015207 (2005)
11. Melde. T., Canton. L., Plessas. W.: Structure of meson-baryon interaction vertices. Phys. Rev. Lett. **102**, 132002 (2008)
12. Amghar, A., Desplanques, B., Theussl, L.: The form-factor of the pion in point form of relativistic dynamics revisited. Phys. Lett. B. **574**, 201-209 (2003)
13. Desplanques, B., Dong, Y. B.: RQM description of PS meson form factors, Constraints from space-time translations, and underlying dynamics. Eur. Phys. J. A. **47**, 13-33 (2011)
14. Allen, T. W., Klink, W. H.: Pion charge form-factor in point form relativistic dynamics. Phys. Rev. C. **58**, 3670-3673 (1998)
15. Allen, T. W., Klink, W. H., Polyzou, W. N.: Point form analysis of elastic deuteron form factors. Phys. Rev. C. **63**, 034002 (2011)
16. Ferraris, M., Giannini, M. M., Pizzo, M., Santopinto, E., Tiator, L.: A three body force model for the baryon. Phys. Lett. B. **364**, 231-238 (1995)
17. Aiello, M., Ferraris, M., Giannini, M. M., Pizzo, M., Santopinto, E.: A three body force model for the electromagnetic excitation of the nucleon. Phys. Lett. B. **387**, 215-221 (1996)
18. Aiello, M., Giannini, M. M., Santopinto, E.: Electromagnetic transition form-factors of negative parity nucleon resonances. J. Phys. G. **24**, 753-762 (1998)
19. De Sanctis, M., Santopinto, E., Giannini, M. M.: A relativistic study of the nucleon form-factors. Eur. Phys. J. A. **1**, 187-192 (1998)
20. De Sanctis, M., Santopinto, E., Giannini, M. M.: A relativistic study of the nucleon helicity amplitudes. Eur. Phys. J. A. **2**, 403-409 (1998)
21. De Sanctis, M., Giannini, M. M., Repetto, L., Santopinto, E.: Proton form-factors in the hypercentral constituent quark model. Phys. Rev. C. **62**, 025208 (2000)
22. Santopinto, E., Giannini, M. M.: Systematic study of longitudinal and transverse helicity amplitudes in the hypercentral constituent quark model. Phys. Rev. C. **86**, 065202 (2012)
23. Bakamjian, B., Thomas, L. H.: Relativistic particle dynamics 2. Phys. Rev. **92**, 1300-1310 (1953)
24. Aznauryan, I. G., et al.: (CLAS Collaboration) Electro-excitation of nucleon resonances from CLAS data in single pion electro-production. Phys. Rev. C. **80**, 055203 (2009)
25. Drechsel, D., Kamalov, S. S., Tiator, L.: Unitary isobar model-MAID 2007. Eur. Phys. J. A. **34**, 69-97 (2007)
26. Bijker, R., Santopinto, E.: Unquenched quark model for baryons: Magnetic moments, spins and orbital angular momenta. Phys.Rev. C. **80**, 065210 (2010)
27. Santopinto, E., Bijker, R.: Flavor asymmetry of sea quarks in the unquenched quark model Phys. Rev. C. **82**, 062202 (2010)
28. Bijker, R., Ferretti, J., Santopinto, E.: $s\bar{s}$ sea pair contribution to electromagnetic observables of the proton in the unquenched quark model Phys.Rev. C. **85**, 035204 (2012)